significant friction (or the surface on which the objects move is not level), at least one object will fail this test, and the system is not functionally isolated.

Third case: *momentarily isolated* systems (where external interactions do not have time to do much)

In the third case, the system participates in strong external interactions that do *not* cancel, but we only look at the system's momentum *just before* and *just after* a very strong and brief interaction. In physics, a **collision** is *any* process in which the internal interaction between two system objects (1) lasts only for a short time but (2) delivers to the objects a much larger impulse than any other interaction during that time. (Note that this definition does *not* require that the objects actually *touch* during a collision!) If the collision process is sufficiently brief, external interactions simply do not have *time* to transfer significant momentum to the system. So, as long as we look at the system just before and just after such a "collision," its total momentum is approximately conserved. We will describe such a system as being **momentarily isolated.**

Exercise C5X.3

Two billiard balls collide on a billiard table. According to the definitions presented above, is the system consisting of the two balls *functionally* isolated, *momentarily* isolated, or isolated because it *floats in space?*

Exercise C5X.4

Two cars skidding on a road hit each other. Can we apply conservation of momentum to this situation? If so, what justifies treating the cars as an isolated system?

C5.3 A Problem-Solving Framework

Part of the aim of this course is to raise your problem-solving skills from the level of predigested "plug-and-chug" problems of the type common in high school physics courses (even advanced-placement courses) to an entirely new level suited to addressing more realistic problems (exemplified by the synthetic and rich-context problems in this text). In this section, I will give you a leg up by describing the general problem-solving "framework" that essentially all experts (but few novices) use to solve serious physics problems, an approach that both research and long experience show to be efficient and effective in generating correct solutions.

This framework involves completing four main tasks: (1) **translating** the problem into mathematical symbols, almost always with the help of a *picture;* (2) building a **conceptual model** of the situation that coherently and logically links together enough physics equations to solve the problem; (3) working out an algebraic **solution** for the quantities to be determined; and (4) **evaluating** (checking) results to see that they make sense. Let us look at each of these tasks in greater detail.

The *translation* step

In the *translation* step, you take the verbal statement of the problem, identify physical quantities of interest, translate those quantities into mathematical symbols so that you can link them to equations, and list the quantities whose values you know. Doing this well prepares you to think more clearly about the conceptual model in the next step and helps you avoid certain beginner errors (such as using the same symbol for different quantities). One *can* do this verbally (e.g., "Let v_i be the ball's initial speed" and so on), but experts know that it is almost always better to define symbols by drawing a schematic *picture* of the situation and labeling the picture's features with those symbols. Pictures are better because (1) human minds are better at processing visual relationships than verbal information, so drawing a picture helps organize one's thinking about a problem; (2) for the same reason, defining symbols visually makes it much easier for both you and your grader to find and remember those definitions; (3) defining reference frame axes (an essential step in any problem involving vectors) is much more easily done on a picture than in words; and (4) defining symbols on a picture *saves time,* since lengthy verbal explanations are not needed.

The *conceptual model* step

In the *conceptual model* step, you wrestle with essential questions such as the following: What theories and/or principles apply here? What simplifications and/or approximations do I have to make to construct a usable model of the situation? What do I need to know to solve the problem, and how can I determine quantities I am not given directly? The goal is to select the equations you need and develop a coherent and physically reasonable plan for using them to solve the problem.

This part is, in my opinion, *by far* the most important part of a problem solution, as it involves about 90% of the thinking you do about the physics of the situation. Yet this is the most important problem-solving skill that typical plug-and-chug problems do not really require and thus do not help you practice. This step is so important at this stage of your education that I will devote the entire next section of this chapter to describing a scheme that makes it easier.

The *algebraic solution* step

Your job in the *algebraic solution* step is take the plan you developed in the previous step and execute it by solving your selected equations *symbolically* for the quantity or quantities of interest. If the problem requests a numerical result, you will *then* plug known values into the symbolic equation and calculate the result, *keeping careful track of units throughout the process.*

It is important to solve your equations symbolically *before* inserting any numbers (other than zeros or simple unitless integers or fractions). Some problems may ask for purely symbolic results, which are often more useful than numerical results because they clearly display how quantities relate to one another. But even if a problem asks for a numerical result, solving equations symbolically is *still* worthwhile because (1) you are *much* less likely to make errors when doing symbolic algebra, (2) using symbols exclusively makes it *much* easier for you or your grader to review your work to find errors, and (3) it *saves time,* because writing a multidigit number with its required units usually takes about 5 to 10 times as much effort of writing a symbol (compare, e.g., writing 2.28×10^6 m/s to writing v_0). Do not make extra work for yourself!

I am *not* saying that you cannot calculate intermediate numerical results in the process of solving a problem: calculating intermediate results can be very helpful when solving complicated problems. What I *am* saying is this: *Never do* algebra *with numbers* (unless they are very simple *unitless* numbers). Observing this simple rule will save you endless grief when solving complicated problems.

The *evaluation* step

The *evaluation* part of the framework is where you thoughtfully assess whether an equation or numerical result *makes sense.* Experts do this kind of assessment *continually* as they work through a problem, but it is essential to do it at the end at least.

The most important way to check a result is to look for *unit consistency:* conceptual and algebraic errors very often lead to unit inconsistencies. For example, if my final expression for a speed is $v = D/\tau^2$ (where D is a distance and τ is a time) or its numerical value has units of meters per second, then I know I have made an error. Long experience has taught me the value of being *constantly* aware of units, yet this powerful tool is consistently underutilized by beginners. I strongly urge you to (1) attach (appropriate) units to *every* numerical quantity you write, (2) keep careful track of units when you calculate, and (3) learn to quickly assess the units implied by symbolic equations.

It is also important to check the *signs* and *magnitudes* of numerical results. For example, if a speed comes out negative or a car ends up with a mass greater than that of a galaxy, there is an error somewhere. Correct numerical results also *tend* to be of the same order of magnitude as known values of the same kind of quantity appearing in the problem. We will discuss methods of assessing the validity of *symbolic* results as we go on in the course.

The importance of practicing these steps on paper

Experts do *some* of these steps in their heads, which is fine when you are an expert. But just as I learned valuable skills as a beginner when my violin teacher made me think through and write out bowing patterns, you will learn empowering thinking habits if you consciously practice this expert problem-solving framework by consciously writing out each step on paper. The worked examples in the rest of this text will illustrate how to use the framework.

C5.4 Constructing Model Diagrams

The conceptual modeling step of this framework is challenging because doing it well involves a number of thinking processes that research suggests beginners find unnatural. For example, experts typically construct a model from the top down (starting with fundamental physical principles and working toward equations) while novices usually work from the bottom up (starting from equations having the variables mentioned in the problem and trying to stitch them together into a coherent whole). Experts are conscious of the physical meaning and limitations of the equations they use; novices tend not to be (and thus end up using inappropriate equations). Experts learn to remember which *symbols* represent known quantities, while novices seek to make this clear by plugging in numbers too early (so that only unknown symbols are left).

This section describes a set of rules for constructing a **conceptual model diagram.** Following these simple rules will automatically walk you through an expertlike thinking process, which in turn will help you solve complicated problems quickly and accurately. The diagram's very structure addresses all the novice difficulties mentioned above.

The rules for constructing a conceptual model diagram

The rules (which assume that you have completed the translation step) are as follows:

1. You will almost always begin a conceptual model diagram by drawing what I call a *helping diagram*. A helping diagram is an abstract representation of the object or system of interest that helps you visualize and think clearly about conceptual aspects of the system (e.g., internal or external interactions) that are not easy to represent in the more realistic diagram you drew for the translation step. Different kinds of problems call for different kinds of helping diagrams: we will discuss the appropriate kind of helping diagram for each type of problem as we go along.
2. Use the helping diagram to construct a *master equation* that expresses the most fundamental physical principle that applies to the problem. If this is a vector equation, write it in *column vector* form, and feel free to use simple definitions (such as $\vec{p} \equiv m\vec{v}$) to express this equation in terms of symbols you have defined in the translation step. (For example, *this* chapter is about applying conservation of momentum, so the master

equation for any problem in this chapter will be a column vector equation expressing conservation of momentum.)

3. Above the equation's equals sign write a brief note explaining what physical principle the equation expresses and *why* it applies in this case. Surround it with a cartoon balloon as if it were spoken by the equals sign. (The explanation can be pretty brief. It might read, "CoM: functionally isolated," short for "this equation describes the principle of conservation of momentum, which applies here because the system is functionally isolated." We will define standard abbreviations such as CoM for important principles as we go along.)
4. Look at every symbol in the equation. (a) If you know that its value is zero or a simple unitless number like 2 or $\frac{1}{2}$, draw an arrow through the symbol and write the value at the arrow's tip. (b) Draw a slash mark through any symbols that cancel on both sides of an equation. (c) Check to see whether the symbol appears on your list of symbols with known values.
5. Circle any symbols whose values remain unknown after step 4. If such a symbol appears more than once in your equations, circle only one instance of the symbol and use lines to link that circle to all other instances of the symbol. Count the circled symbols, and compare to the number of equations. (Count each meaningful row of a vector equation as a *separate* equation. A row that says $0 + 0 = 0$ is *not* meaningful.) If you have at least as many independent equations as unknowns, you should be able to solve the problem, and your model is complete.
6. Otherwise, consider a different equation containing one or more of your unknowns, carefully assessing whether it applies in this situation (or whether making a reasonable approximation or assumption would *allow* it to apply). If it does, write the equation below the master equation and draw a line connecting any symbol appearing in the new equation with the same symbol in previous equations.
7. Repeat, starting at step 3, until you have enough equations to solve. In step 3, the equivalence sign $\equiv$ is sufficient explanation for definitions, but you should attach an explanatory cartoon balloon to the equals sign of any other equations.

Occasionally (most often in rich-context problems), you may need to make an estimate to determine a symbol's value. If so, treat the equation linking the symbol to its estimated value as just another equation in step 6, and write "estimated" (along with any appropriate explanation if the estimation is not trivial) in the equation's cartoon balloon. You may also encounter known or unknown quantities that ultimately prove to be irrelevant. Keep focused on solving for the unknowns that the problem asks you to solve.

You may also need to be flexible in adapting the above rules to specific problems. If you do something outside the rules, simply explain what you are doing.

I also find it very helpful to express vector components (whenever possible) in terms of symbols representing intrinsically *positive* numbers (such as vector magnitudes or distances). This means that any minus signs associated with the components get displayed explicitly in the equation instead of being "buried" inside the symbol. I find that this markedly reduces the number of sign errors I make.

If you complete such a diagram well, (1) you will always know which symbols are known and unknown, (2) you will know without doing any algebra that your model is complete, (3) your "equals sign" notes will have provided sufficient physical reasoning to support your model, and (4) your diagram will essentially tell you how to solve the problem mathematically. Most importantly, though, you will have followed an expertlike reasoning process that focuses first on important principles and depends on *physical* reasoning to fill in the details.

You *can* write your conceptual model step in *prose* instead of constructing a diagram; this is sometimes easier for simple problems. But if you do write a prose model, make sure that you follow a thinking process like that you would use to build the diagram, and use complete sentences to describe the equations you will use and why they apply in this situation (or what approximation you are making that allows them to apply). Examples in this text will typically provide model steps in both diagram and prose form. Follow your instructor's advice about which approach to use for your homework solutions. (I personally recommend that you use conceptual model diagrams for any homework problem whose number is marked with an asterisk.)

C5.5 Solving Conservation of Momentum Problems

The helping diagram for conservation of momentum problems

The conceptual model diagram for any problem in *this* chapter will begin with a helping diagram that we will call an **interaction diagram.** You can construct an interaction diagram as follows. Draw a large circle to represent the system, and draw one rectangular box inside that circle for each object inside the system, labeling the box with the object's name. If the objects inside the system interact with objects outside the system, draw a box outside the circle for each relevant object outside the system, and label these boxes as well. Then draw lines connecting the boxes to represent the internal and external interactions between these objects, and label these lines to indicate the type of interaction involved. (If the system floats in space, simply write "floats in space" along the margin of the circle and do not include any external gravitational interactions.)

An interaction diagram helps you clearly visualize the system and sharply distinguish between external and internal interactions. In conservation of momentum problems we are especially interested in understanding the *external* interactions, so that we know what approximations we have to make to say that the system is isolated.

The master equation for conservation of momentum problems

The master equation for any problem in this chapter will express the law of conservation of momentum (CoM), which states that a suitably isolated system's total momentum before some process of interest is equal to its total momentum after that process:

$$\vec{p}_{1i} + \vec{p}_{2i} + \vec{p}_{3i} + \cdots = \vec{p}_{1f} + \vec{p}_{2f} + \vec{p}_{3f} + \cdots \qquad \text{(C5.1)}$$

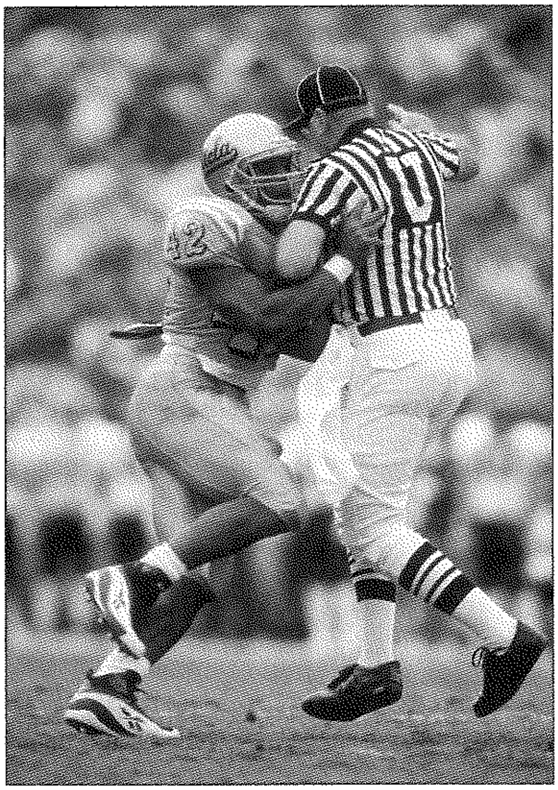
where the subscripts 1, 2, 3, ... refer to the objects in the system and subscripts i and f mean initial and final, respectively. Adapt this equation for the number of objects in the system in question, use the definition $\vec{p} \equiv m\vec{v}$ to rewrite the momenta in terms of masses and velocities, and write it out in column vector form. Your explanation for the equals sign in this equation should include this principle's abbreviation **CoM** followed by some statement of how this system is isolated (using the categories discussed in section C5.2).

The *translation* step for conservation of momentum problems

The *translation* step for any conservation of momentum problem should include all items on the following checklist:

1. *Two* pictures that show the system in its initial and final states, respectively.

C5S.2 A 110-kg football player running at 3 m/s collides head-on with a 55-kg referee by accident. This collision gives an impulse to the referee at the expense of the player. Irrespective of how *large* the impulse is, how will the magnitude of the change in the referee's velocity during the collision compare with that of the player? Explain carefully.



C5S.3 Imagine that someone places you at rest on a flat, utterly frictionless surface. You cannot walk to the edge of the surface, because your shoes will not grip it. Is there another way to use those shoes and the law of conservation of momentum to get off the surface? Explain your solution.

C5S.4 Imagine that a rocket is launched from an asteroid in deep space and fires its engines until the speed of the rocket relative to the asteroid is equal to the speed of the rocket's exhaust. The exhaust ejected by the engine is now at *rest* with respect to the asteroid. If the engines continue to fire, will the rocket's speed with respect to the asteroid still increase? (If it does, note that the exhaust will now move in the same direction as the rocket relative to the asteroid.) Explain whether the rocket can still go faster from the point of view of an observer on the asteroid.

*C5S.5 Two hockey pucks, one with mass m and the other with mass $2m$, slide on a flat, frictionless plane of ice. Originally, the lighter puck is sliding in a direction 30° west of south at a speed of 3 m/s while the more massive puck is sliding in a direction 60° north of east at 1.5 m/s. The pucks collide and stick together. What is their joint velocity (magnitude and direction) after the collision?

*C5S.6 Two identical hockey pucks slide on a flat, frictionless plane of ice. Originally, one is sliding in a direction 60° south of west at a speed of 2 m/s, while the other is sliding in a direction 30° west of north at a speed of 2 m/s. The pucks collide and stick together. What is their joint velocity (magnitude and direction) after the collision?

*C5S.7 Two people slide on a frictionless, flat, horizontal plane of ice. Person A, whose mass is 54 kg, is sliding due east at a speed of 2.5 m/s. Person B, whose mass is 68 kg, is sliding due south at a speed of 1.8 m/s. These people collide and hold on to each other. What are the magnitude and direction of their joint velocity after the collision?

*C5S.8 In the All-Alaska Ice Floe Softball finals, the right fielder for the Nome IceSox (who is floating on a small chunk of ice in still water) makes an outstanding catch of a line drive. If the combined mass of the fielder and the ice is 540 kg, and it was traveling due north at 0.15 m/s (due to the fielder's frenzied paddling) before the catch, and the ball has a mass of 0.25 kg and is traveling at 32 m/s due east when caught, what is the final heading of the fielder just after the ball is caught?

*C5S.9 A pontoon boat (weight 1200 lb) sits at rest on a still lake near a dock. Your friend Dana, whose weight is 160 lb, runs off the end of the dock at a speed of 15 mi/h and jumps onto the deck of the boat. Dana does not know anything about physics and so is surprised that the boat ends up moving away from the dock. How fast is it moving after Dana lands?

*C5S.10 During the filming of a certain movie scene, the director wants a small car (mass 750 kg) traveling due east at 35 m/s to collide with a small truck (mass 3200 kg) traveling due north. The director also wants the collision to be arranged so that just afterward the interlocked vehicles travel straight toward the camera (which is placed at a safe distance, of course). If the line between the camera and the collision makes an angle of 29° with respect to north, at what speed should the trucker drive?

Rich-Context

*C5R.1 A small asteroid of mass 2.6×10^9 kg is discovered traveling at a speed of 18 km/s on a direct heading for Starbase Alpha, which is in deep space well outside the solar system. Lacking weapons of sufficient power to destroy the asteroid, the frightened starbase inhabitants decide to deflect it by hitting it with a remote-controlled spaceship. The spaceship has an empty mass of 25,000 kg and a top speed of 85 km/s

potential energy to be zero if the rock is at ground level. A person standing at the bottom of a well throws the rock vertically upward from 20 m below ground level. The rock makes it all the way up to 1 m below ground level before falling back into the well. The total energy of the rock-earth system is
A. Negative.
B. Zero.
C. Positive (in this particular case).
D. Positive because energy is *always* positive.
E. The answer depends on the rock's mass.
F. The answer depends on the rock's initial speed.
T. The answer depends on something else (specify).

HOMEWORK PROBLEMS

Basic Skills

C6B.1 A car is traveling north at 30 mi/h. A truck having 4 times the mass of the car is traveling at 60 mi/h west. How many times greater is the truck's kinetic energy than the car's? Explain your reasoning.

C6B.2 A typical arrow might have a mass of 100 g and move at a speed of about 100 m/s. How does its kinetic energy compare to that of person weighing 110 lb running at a speed of 8.8 mi/h?

C6B.3 Consider an object interacting gravitationally with the earth. If we move the object from vertical position A to vertical position B, we find that the system's gravitational potential energy *increases* by 10 J. If we move it from vertical position B to vertical position C, the system's potential energy *decreases* by 5 J. If we take the system's reference separation to be when the object is at position B, what is the system's potential energy when the object is at each of the three points?

C6B.4 Consider a 5-kg object interacting gravitationally with the earth. Imagine that we set up a standard reference frame with the z axis pointing vertically upward. If the interaction's potential energy when the object is at $z = 5$ m is -50 J, what is the approximate z-position of the object when it is at its reference separation from the earth?

C6B.5 Consider a 0.25-kg ball interacting gravitationally with the earth. Imagine that we set up a reference frame in standard orientation on the earth's surface and define ground level to be the reference separation and set $z = 0$ there. Imagine that a person throws the ball upward into the air and that as the ball leaves the person's grasp 2.0 m above the ground, it has a speed of 12 m/s. What is the *system* involved here, and what is its total energy at this time?

C6B.6 Consider a 0.20-kg ball interacting gravitationally with the earth. Imagine that we set up a reference frame in standard orientation on the earth's surface and define ground level to be the reference separation and set $z = 0$ there. Imagine that a person at the bottom of a well throws the ball upward and that when the ball leaves the person's grasp 8.0 m below the ground, it has a speed of 6 m/s. What is the system's total energy at this time?

Synthetic

The starred problems are especially well suited for practicing the use of the problem-solving framework and conceptual model diagrams.

C6S.1 A 1000-kg car travels down a road at 25 m/s (55 mi/h). What is its kinetic energy? Now imagine that the car's speed increases to 35 m/s (77 mi/h), which is 40% faster. Is the kinetic energy 40% larger or not? (Note that the severity of a crash is roughly proportional to the kinetic energy that participants bring to it.)

C6S.2 Imagine that if you drop an object from a certain height, its final speed is 20 m/s when it reaches the ground. If you throw the object vertically downward from the same height with an initial speed of 20 m/s, will its final speed be 40 m/s? Carefully explain why or why not.

*C6S.3 A 2.0-kg coconut (initially at rest) falls from the top of a coconut tree 15 m high. What is the coconut's kinetic energy when it hits the ground? What is its speed?

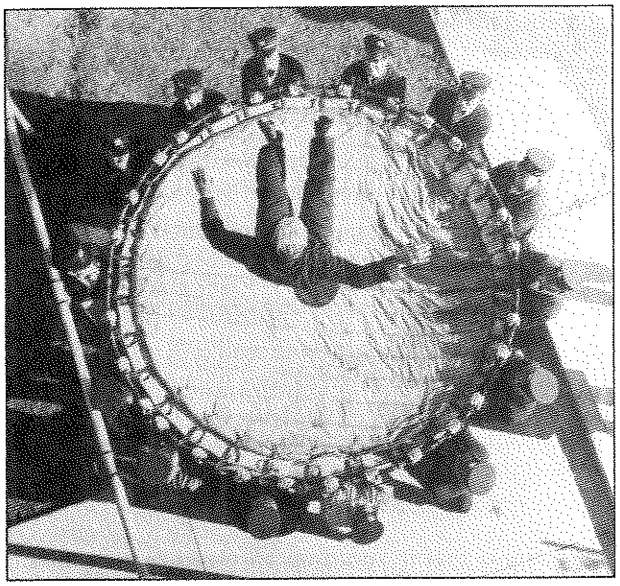
*C6S.4 If a person wanting to dive from a seaside cliff does not feel safe hitting the water faster than 20 m/s (44 mi/h), what is the maximum height from which he or she should dive?

*C6S.5 Imagine that you are standing at the top of a cliff 45 m high overlooking the ocean and you throw a rock straight downward at a speed of 15 m/s. What is the rock's speed when it hits the water?

*C6S.6 Imagine that you are throwing a tennis ball at a Frisbee lodged in a tree 15 m above the ground. If you want the ball's speed to be at least 5 m/s when it hits the Frisbee, what should its speed be as it leaves your hand? Does your answer depend on the angle that the ball's velocity makes with the horizontal when it leaves your hand?

Rich-Context

*C6R.1 You are designing a safety net for use by firefighters that can safely catch a person jumping from the top of a 30-story building. What will be the person's approximate speed when hitting the net? How much kinetic energy will the net have to convert safely to other forms? (Make appropriate estimates.)



C6R.2 In July 1994, about 20 fragments of comet Shoemaker-Levy struck the planet Jupiter, each traveling at a final speed of roughly 60 km/s. These impacts were closely studied because they promised to be the most cataclysmic impacts ever witnessed. No one knew exactly *how* cataclysmic, though, because the fragments' sizes (and thus masses) were too small to measure. One estimate of the total energy released by fragment G's impact was 4×10^{22} J (equivalent to the detonation of roughly 100 million typical atomic bombs). Use *this* to estimate fragment G's size, first assuming first that it was solid rock and then that it was solid ice, which have densities of about 3000 kg/m^3 and 920 kg/m^3, respectively. Don't worry about being excessively precise. (This illustrates how even a little knowledge about kinetic energy can help answer questions about objects that can barely be *seen* by the best telescopes!)

An infrared image of the fireball created when fragment G hit Jupiter. The energy released by the impact can be estimated from images like this.

ANSWERS TO EXERCISES

C6X.1 According to chapter C3, $1\,\text{N} \equiv 1\,\text{kg}\cdot\text{m/s}^2$, so

$$1\,\text{J} = 1\,\cancel{\text{J}}\left(\frac{1\,\text{kg}\cdot\cancel{\text{m}^2}/\cancel{\text{s}^2}}{1\,\cancel{\text{J}}}\right)\left(\frac{1\,\text{N}}{1\,\cancel{\text{kg}}\cdot\cancel{\text{m}}/\cancel{\text{s}^2}}\right) = 1\,\text{N}\cdot\text{m} \tag{C6.19}$$

C6X.2 The person's kinetic energy is $\frac{1}{2}(50\,\text{kg})(3\,\text{m/s})^2 = 225\,\text{kg}\cdot\text{m}^2/\text{s}^2 = 225\,\text{J}$, while the car's kinetic energy is $\frac{1}{2}(1000\,\text{kg})(30\,\text{m/s})^2 = 450{,}000\,\text{kg}\cdot\text{m}^2/\text{s}^2 = 450{,}000\,\text{J}$.

C6X.3 According to the inside front cover, the earth's mass is about 6×10^{24} kg. Therefore equation C6.6 implies that

$$\frac{K_{\text{earth}}}{K_{\text{truck}}} = \frac{m_{\text{truck}}}{m_{\text{earth}}} = \frac{6 \times 10^3\,\cancel{\text{kg}}}{6 \times 10^{26}\,\cancel{\text{kg}}} = 10^{-23} \tag{C6.20}$$

Therefore, the earth's kinetic energy is about $10^{-23}(10^5\,\text{J}) = 10^{-18}\,\text{J}$, which is immeasurably small compared to the truck's energy.

C6X.4 When the two objects have comparable mass (e.g., like repelling magnetic carts on a track), the positions of *both* carts change appreciably during the